

CODEFORCES REVIEW

CONTENTS

2013D - 1900	I
2014H - 1900	I
2035D - 1800	I
2009G1 - 1900	2
2002D1, 2021C2 - 1900	2
1991E - 1900	3
1991D - 1900	3
2039C2 - 1800	3

.1 2013D - 1900

Problem 1. ¹ Given an array A and an operator p that for each a_i and a_{i+1} where $i \geq 1$, p may decrease $|a_i - a_{i+1}|$. p can be performed by infinite times and find the minimum $|\max_{i=1}^n(a_i) - \min_{i=1}^n(a_i)|$

Solution 1. Suppose b_i is the result of performing p on a_i given the prefix of array A $\{a_1, a_2, \dots, a_i\}$. Create a stack S to load these b_i and the count of it c_i . Then $\{a_1, a_2, \dots, a_i\}$ is stored in S as $(b_1, c_1), (b_2, c_2), \dots, (b_m, c_m)$. And we keep the pairs in ascending order of b_i . For each new a_{i+1} , merge it to the top if $a_{i+1} < b_m$ and then merge the top downwards until $b_k > b_{k-1}$. This way, each a_i is loaded in the array for once, the time of merging until a_i is at most i , so the time complexity is $O(n)$.

.2 2014H - 1900

Problem 2. ² Given an array A , check for each a_i , if there exists $A_m = \{a_{m_1}, a_{m_2}, \dots, a_{m_j}\}$ s.t. $a_i \in A_m$ and j is even.

Solution 2. If the size of A is small, for instance, $1e6$, then first hashing the A in a much bigger set, for instance, $\{1, 2, \dots, 2^{64}\}$, and check if the xor sum of the array is 0. The hash is very important, without which, several different numbers can also get xor sum 0 (e.g., $\{1, 2, 3\}$). The possibility of reaching such bad situation after hashing is $\frac{1}{2^{64}}$.

.3 2035D - 1800

Problem 3. ³ Given an array A and an operator p that for each a_i and a_j that $i < j$, p can update a_i to $a_i \gg 1$ and a_j to $a_j \ll 1$. p can

¹<https://codeforces.com/problemset/problem/2013/D>

²<https://codeforces.com/problemset/problem/2014/H>

³<https://codeforces.com/problemset/problem/2035/D>

be performed by infinite times and find the maximum sum of all the prefixes of A .

Solution 3. A little similar to 2013D. In 2013D, we store b_i and c_i , the information of a_i after p in a stack S . This approach is applicable in this problem too. In this problem $b_i = \min\{b_i^j | b_i^j \ll c = a_i\}$, and c_i is the sum of all c_k that $k < i$ that can reach the maximum prefix sum from 1 to i . This sum is obtained by merging from the top of the stack downwards. If $c_k (k < i)$ is added to c_i , then b_k is popped out from the stack and added to the final sum. After the merging is terminated, we push (b_i, c_i) onto S . Each (b_i, c_i) pair is at most pushed to S by once and popped by once. So the time complexity is $O(n)$

.4 2009G1 - 1900

Problem 4. ⁴ Given an array A and an operator op that can change any element a_i to any another value. Find the minimum step required to make A a consecutive array in which $a_{i+1} - a_i = 1$ for any i .

Solution 4. Create an array B that $b_i = a_i - i$. Count the frequency f_i of each b_i and the answer is $A.size() - \max(f_i)$.

.5 2002D1, 2021C2 - 1900

Problem 5. ⁵⁶ Continuously query about a property P with an update before each query.

Solution 5. Find another easy-to-maintain property P' s.t. if P' is satisfied then P is satisfied. The time complexity of checking for P' after each update is $O(1)$ or $O(\log n)$.

⁴<https://codeforces.com/problemset/problem/2009/G1>

⁵<https://codeforces.com/problemset/problem/2002/D1>

⁶<https://codeforces.com/problemset/problem/2021/C2>

.6 1991E - 1900

Takeaway 1. ⁷ If I want to use two colors to color an undirected graph G , in which for every v_1, v_2 connected by edge E , their colors are different, the only thing to judge if I can make the coloring is check if the graph is bipartite.

.7 1991D - 1900

Takeaway 2. ⁸ If $a - b = kx$, and x is the power of 2, then $a \oplus b = x$ where \oplus means XOR

.8 2039C2 - 1800

Takeaway 3. ⁹ Some basic math that is useful for this kind of CF problems. Suppose $a \geq b$, then $a - b \leq a \oplus b \leq a + b \leq 2a$ since \oplus is just addition without carry or subtraction without borrowing. Suppose $a > b$, then $\text{lcm}(a, b) \leq 2\max(a, b) < 2a$.

⁷<https://codeforces.com/problemset/problem/1991/E>

⁸<https://codeforces.com/problemset/problem/1991/D>

⁹<https://codeforces.com/problemset/problem/2039/C2>